

SIR VEN

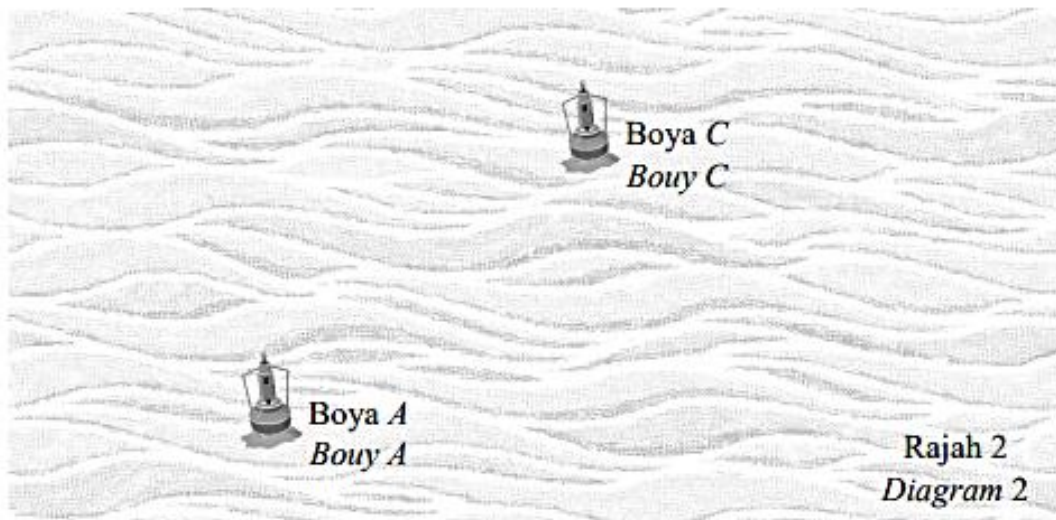
(GURU ADIWIRA KEBANGSAAN 2019)

SPM 2023

SOALAN RAMALAN
MATEMATIK TAMBAHAN
KERTAS 2

KOORDINAT GEOMETRI
COORDINATE GEOMETRY





Rajah 2 menunjukkan kedudukan dua boya pengesan tsunami yang diletakkan di permukaan lautan iaitu boya A dan boya B . Dua lagi boya baharu akan ditambah pada sistem pengesan tersebut. Jika diwakilkan pada satah cartes, koordinat boya A dan boya C masing-masing ialah $(1,3)$ dan $(7,7)$. Mengikut rancangan, boya B akan diletakkan paksi- y dan berjarak sama dari boya A dan boya C . Boya D juga akan ditambah dengan keadaan $ABCD$ sebuah rombus.

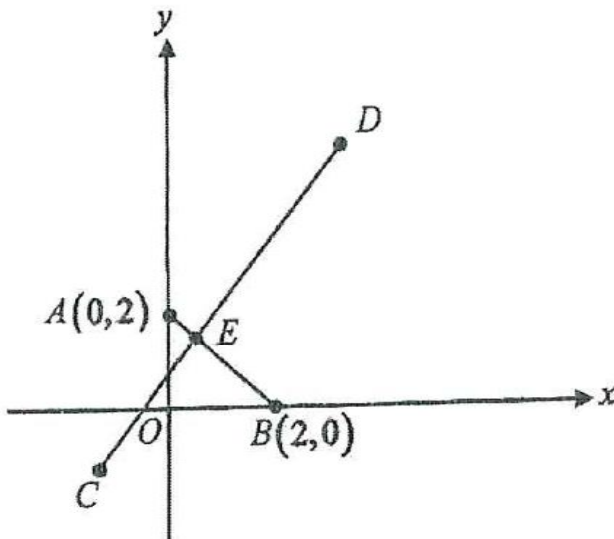
Diagram 2 shows the position of two tsunami detection buoys placed on the surface of the ocean, namely buoy A and buoy B . Two more new buoys will be added to the detection system. If represented on the cartesian plane, the coordinates of buoy A and buoy C are $(1,3)$ and $(7,7)$ respectively. According to the plan, buoy B will be placed on the y -axis and be the same distance from buoy A and buoy C . Buoy D will also be added, such that $ABCD$ is a rhombus.

Cari

Find

- (a) koordinat B dan koordinat D , [5 markah]
coordinates of B and of D , [5 marks]
- (b) jika luas rombus $ABCD$ ialah 52 km^2 . Cari jarak terdekat A ke BC .
if the area of rhombus $ABCD$ is 52 km^2 . Find the closest distance A to BC . [3 markah]
[3 marks]

Rajah 4 menunjukkan garis lurus AB bersilang dengan garis lurus CD pada titik E.
Diagram 4 shows the straight-line AB intersects with straight line CD at point E.



Rajah 4
Diagram 4

Persamaan garis lurus CD ialah $-3x + \sqrt{2}y = 1$

The equation of straight-line CD is $-3x + \sqrt{2}y = 1$

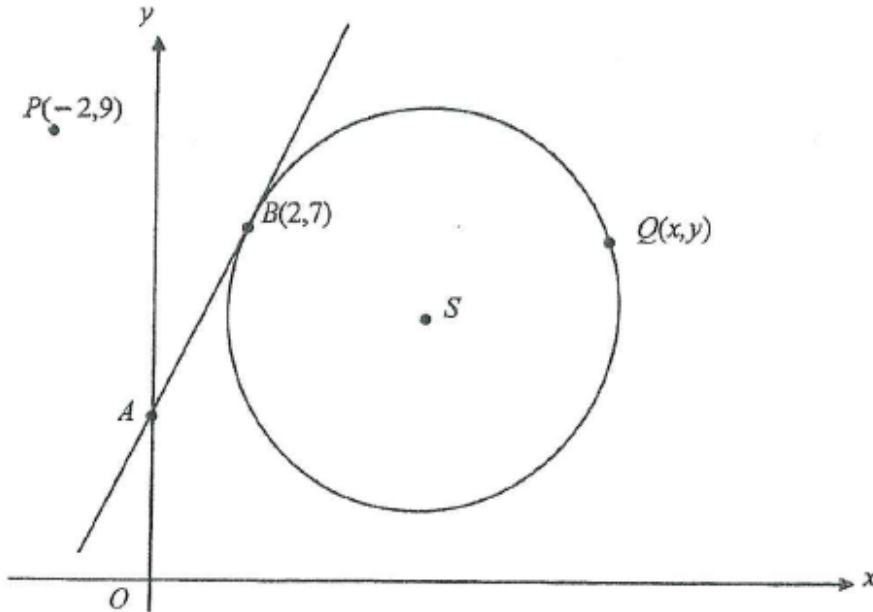
a) Cari koordinat titik E dalam bentuk $(p + q\sqrt{2}, r + s\sqrt{2})$

Find the coordinates of point E in the form $(p + q\sqrt{2}, r + s\sqrt{2})$

b) Diberi jarak antara titik E dan titik $(-1 + m\sqrt{2}, -\sqrt{2})$ adalah $\sqrt{17}$. Cari nilai-nilai m

Given the distance between point E and point $(-1 + m\sqrt{2}, -\sqrt{2})$ is $\sqrt{17}$. Find the values of m.

Rajah 2 menunjukkan satu titik Q yang bergerak di sepanjang lilitan sebuah bulatan dengan pusat S. Garis lurus $2x = y - 3$ ialah tangen kepada bulatan itu pada titik B dan bersilang pada paksi-y di titik A. *Figure 2 shows a point Q moving along the circumference of a circle with center S. The straight line is $2x = y - 3$ tangent to the circle at point B and intersects the y-axis at point A.*



- a) Titik S ialah imej bagi titik P di bawah satu pantulan pada garis $2x = y - 3$ s. Titik-titik P, B dan S adalah segaris. Cari persamaan lokus bagi titik Q

Point S is the image of point P under a reflection at line $2x = y - 3$. The points P, B and S are in a collinear. Find the equation of the locus of point Q.

- b) Jika garis lurus BS dipanjangkan kepada titik N, dengan keadaan $3BN=4SN$, hitung luas sisi empat OABN.

If the straight-line BS is extended to point N, such that $3BN=4SN$, calculate the area of quadrilateral OABN.



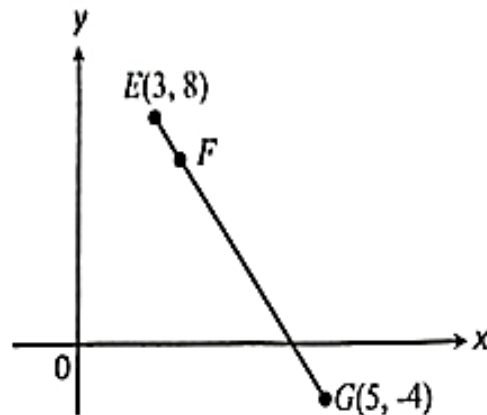
Titik S bergerak dengan keadaan sentiasa sama jarak dari $J(-8,0)$ dan $K(0,4)$. Titik Q pula bergerak supaya jaraknya dari titik $R(4,6)$ adalah sentiasa 10 unit.

A point S moves such that it is always equidistance from $J(-8,0)$ and $K(0,4)$. While a point Q moves such that its distance from point $R(4,6)$ is always 10 units.

- (a) Cari persamaan lokus titik S . [2 markah]
Find the equation of the locus of S . [2 marks]
- (b) Tunjukkan bahawa lokus Q ialah $x^2 + y^2 - 8x - 12y - 48 = 0$. [2 markah]
Show that the locus of Q is $x^2 + y^2 - 8x - 12y - 48 = 0$. [2 marks]
- (c) Hitung titik persilangan di antara dua lokus itu. [4 markah]
Find the point of intersections between the two loci. [4 marks]

Rajah 11 menunjukkan garis lurus EFG yang menyalang paksi- x . Persamaan garis lurus EFG ialah $y = -6x + 26$.

Diagram 11 shows a straight line of EFG that intersects the x -axis. The equation of the straight line EFG is $y = -6x + 26$.



Rajah 11 / Diagram 11

- (a) Cari persamaan garis lurus yang berserenjang dengan garis lurus EFG dan melalui titik E .
Find the equation of a straight line perpendicular to the straight line EFG and passes through point E .
[2 markah]
[2 marks]
- (b) Diberi bahawa titik F membahagi tembereng lurus EG dengan keadaan $EF : FG = 1 : 3$. Cari koordinat bagi F .
It is given that the point F divides the straight line EG such that $EF : FG = 1 : 3$. Find the coordinates of F .
[2 markah]
[2 marks]
- (c) Cari luas segi tiga EOF .
Find the area of triangle EOF .
[3 markah]
[3 marks]
- (d) Cari persamaan lokus bagi titik N yang bergerak dengan keadaan jaraknya dari titik G sentiasa 6 unit.
Find the equation of the locus of point N which moves such that its distance from point G is always 6 units.
[3 markah]
[3 marks]



Lengkung $y = x^3 - 3x^2 - 9x + 5$ melalui titik $A(0,5)$ dan mempunyai dua titik pusingan $P(3,-2)$ dan Q .

The curve $y = x^3 - 3x^2 - 9x + 5$ passes through point $A(0,5)$ and has two turning points $P(3,-2)$ and Q .

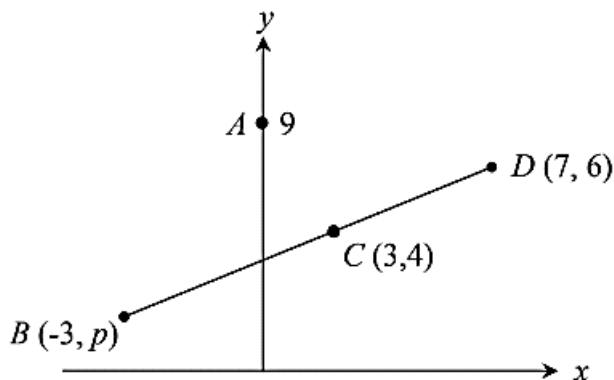
Cari

Find

- (a) kecerunan lengkung itu pada titik A . [2 markah]
the gradient of the curve at point A . [2 marks]
- (b) koordinat titik Q dan tentukan sama ada Q adalah titik maksimum atau titik minimum. [5 markah]
the coordinates of point Q and determine whether Q is the maximum point or the minimum point. [5 marks]
- (c) peratus perubahan dalam y jika terdapat perubahan kecil dalam x sebanyak 3% apabila $x=5$. [3 markah]
the percentage change in y if there is a small change in x by 3% when $x=5$. [3 marks]

Rajah 4 menunjukkan lokasi empat buah lampu isyarat A , B , C dan D di atas satah Cartes.

Diagram 4 shows the positions of four traffic lights A , B , C and D on the Cartesian plane.

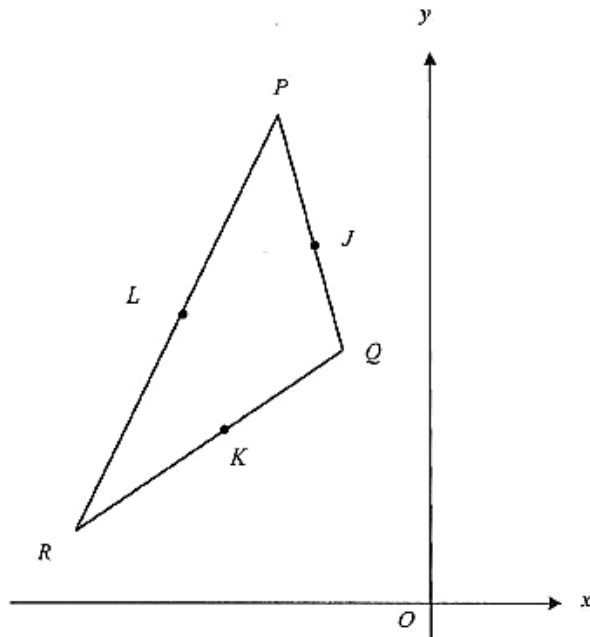


Rajah 4
Diagram 4

Diberi bahawa $B(-3, p)$, $C(3, 4)$, dan $D(7, 6)$ adalah segaris dengan keadaan $BC:CD = m : n$.

Given that $B(-3, p)$, $C(3, 4)$, and $D(7, 6)$ are collinear such that $BC : CD = m : n$.

- (a) Tentukan $m : n$. [3 markah]
Determine $m : n$. [3 marks]
- (b) Seterusnya, cari nilai p . [2 markah]
Hence, find the value of p . [2 marks]
- (c) Cari luas, dalam unit^2 , bagi $\triangle ABD$. [2 markah]
Find the area, in units^2 , of $\triangle ABD$. [2 marks]



Rajah 1
Diagram 1

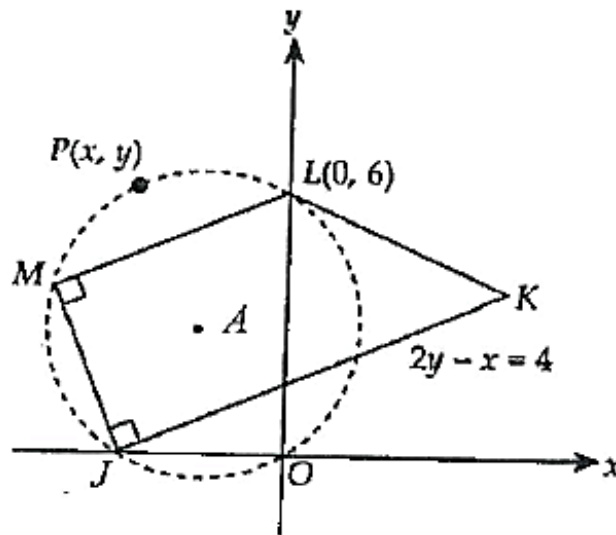
Diberi bahawa $J(-2,12)$, $K\left(\frac{-9}{2},6\right)$, dan $L(-5,10)$ adalah masing-masing titik tengah bagi garis lurus PQ , QR , dan RP dengan keadaan $JQKL$ membentuk segi empat selari.

It is given that $J(-2,12)$, $K\left(\frac{-9}{2},6\right)$, and $L(-5,10)$ are midpoint of straight line PQ , QR , and RP respectively such that $JQKL$ formed a parallelogram.

- (a) Cari persamaan garis lurus PQ . [2 markah]
Find the equation of straight line PQ . [2 marks]
- (b) Garis lurus PQ dipanjangkan sehingga bersilang dengan paksi-y pada titik M .
Cari luas bagi segi tiga JMK . [3 markah]
Straight line PQ is extended and intersect y -axis at a point M .
Find the area of triangle JMK . [3 marks]
- (c) Suatu titik S bergerak dengan keadaan jaraknya dari titik M adalah sentiasa dua kali jaraknya dari titik J .
Cari persamaan lokus S . [3 markah]
A point S moves such that its distance from point M is always twice its distance from point J .
Find the equation of the locus of S . [3 marks]

Dalam Rajah 7, $JKLM$ ialah sebuah trapezium dan bulatan berpusat A .

In Diagram 7, $JKLM$ is a trapezium and a circle centered at A .



Rajah 7

Diagram 7

- (a) Persamaan garis lurus JK ialah $2y - x = 4$ dan koordinat titik L ialah $(0, 6)$.

The equation of the straight line JK is $2y - x = 4$ and the coordinates of point L is $(0, 6)$

Cari

Find

- (i) persamaan garis lurus JM ,
the equation of the straight line JM ,
- (ii) koordinat titik M .
The coordinate of point M .

[5 markah]

[5 marks]

- (b) $P(x, y)$ bergerak di sepanjang lilitan bulatan yang melalui titik J dan titik L dengan keadaan JL ialah diameter bulatan itu. Cari persamaan lokus bagi P . [3 markah]

$P(x, y)$ moves during the circumference of a circle which passes through the point J and point L such that JL is the diameter of the circle. Find the equation of locus of P .

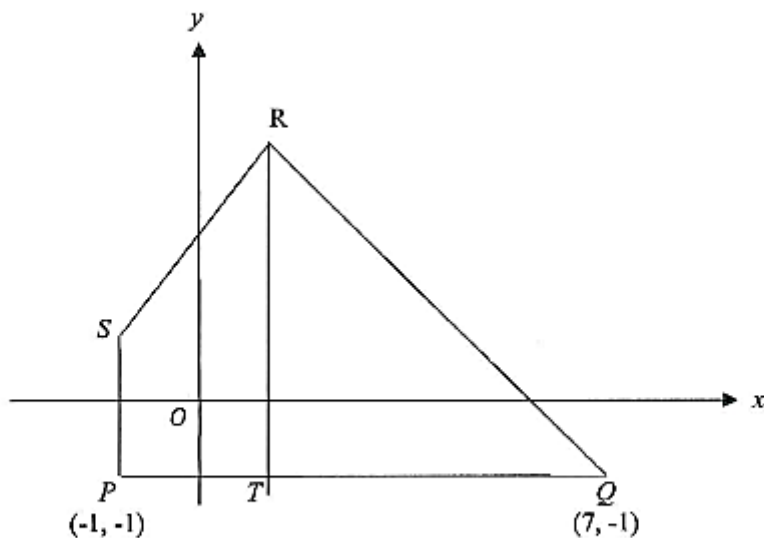
[3 marks]

- (c) Diberi koordinat $K(4, 4)$, cari luas trapezium $JKLM$. [2 markah]

Given the coordinate of $K(4, 4)$, find the area of trapezium $JKLM$. [2 marks]

Rajah 2 menunjukkan sebuah sisi empat $PQRS$.

Diagram 2 shows a quadrilateral $PQRS$.



Rajah 2

Diagram 2

Diberi bahawa $PQ = 4PT$, $PS = PT$ dan $TR = TQ$. Garis lurus PS dan garis lurus TR adalah selari dengan paksi- y .

Given that $PQ = 4PT$, $PS = PT$ and $TR = TQ$. The straight line PS and the straight line TR are parallel to y -axis.

(a) Satu titik W bergerak melalui titik S dan titik T dengan berpusatkan titik P .

Cari persamaan bagi pergerakan titik W .

[3 markah]

A point W moves through point S and point T centered at point P .

Find the equation for the movement of point W .

[3 marks]

(b) Garis lurus RS ialah pembahagi dua sama serenjang bagi garis lurus yang menyambungkan $A(4,6)$ dan $B(n,t)$.

Cari nilai n dan nilai t .

[5 markah]

The straight line RS is the perpendicular bisector of the straight line joining $A(4,6)$ and $B(n,t)$.

Find the value of n and of t .

[5 marks]

YIK

$$\text{Midpoint } AC = \left(\frac{7+1}{2}, \frac{7+3}{2} \right) \text{ atau } m_{AC} = \frac{7-3}{7-1}$$

$$m_{BD} = \frac{y-5}{0-4} = -\frac{3}{2}$$

$$B(0,11)$$

$$\frac{0+x}{2} = 4 \text{ atau } \frac{y+11}{2} = 5$$

$$D(8,-1)$$

$$BC = \sqrt{(0-7)^2 + (11-7)^2}$$

$$\frac{52}{2} = \frac{1}{2} \times \sqrt{65} \times t$$

$$t = 6.450$$

MRSM

(a) $x+y=2$ @ $y=-x+2$ @ $x=-y+2$
 $-3(2-y)+\sqrt{2}y=1$ @ $-3x+\sqrt{2}(2-x)=1$
 $y = \frac{7}{3+\sqrt{2}} \times \left(\frac{3-\sqrt{2}}{3-\sqrt{2}} \right)$ @ $x = \frac{2\sqrt{2}-1}{3+\sqrt{2}} \times \left(\frac{3-\sqrt{2}}{3-\sqrt{2}} \right)$
 $y = \frac{21-7\sqrt{2}}{9-2}$ @ $x = \frac{-3+\sqrt{2}+6\sqrt{2}-2(2)}{9-2}$

$$(-1+\sqrt{2}, 3-\sqrt{2})$$

(b) $\sqrt{((-1+\sqrt{2})-(-1+m\sqrt{2}))^2 + ((3-\sqrt{2})-(-\sqrt{2}))^2} = \sqrt{17}$
 $\sqrt{2} \times \sqrt{2} = 2$
 $2m^2 - 4m - 6 = 0$
 $m = 3, m = -1$

MRSM

(a) (6,5)

$$\sqrt{(*6-2)^2 + (*5-7)^2} \text{ @ } \sqrt{(-2-2)^2 + (9-7)^2}$$

@

$$\sqrt{(x-*6)^2 + (y-*5)^2}$$

$$\sqrt{(-2-2)^2 + (9-7)^2} = \sqrt{(x-*6)^2 + (y-*5)^2}$$

@

$$\sqrt{(*6-2)^2 + (*5-7)^2} = \sqrt{(x-*6)^2 + (y-*5)^2}$$

@

$$\sqrt{20} = \sqrt{(x-*6)^2 + (y-*5)^2}$$

$$x^2 + y^2 - 12x - 10y + 41 = 0$$

(b)

$$\frac{2(3)+x(1)}{1+3} = *6 \text{ @ } \frac{7(3)+y(1)}{1+3} = *5$$

$$(18,-1)$$

$$A = \frac{1}{2} [0(3)+0(7)+2(*-1)+*18(0)] - [0(0)+2(3)+*1$$

SABK S3

3 (a) $SJ=SK$
 $\sqrt{(x+8)^2 + y^2} = \sqrt{x^2 + (y-4)^2}$
 $2x + y + 6 = 0$

3 (b) $\sqrt{(x-4)^2 + (y-6)^2} = 10$
 $x^2 - 8x + 16 + y^2 - 12y + 36 = 100$
 $x^2 + y^2 - 8x - 12y - 48 = 0$

3 (c) $2x + y + 6 = 0$
 $y = -2x - 6$ -----(1)
 $x^2 + y^2 - 8x - 12y - 48 = 0$ ----- (2)

Gantikan (1) ke dalam (2)

$$x^2 + (-2x-6)^2 - 8x - 12(-2x-6) - 48 = 0$$

$$x^2 + 8x + 12 = 0$$

$$(x+6)(x+2) = 0$$

$$x = -6, x = -2$$

Koordinat titik persilangan (-6,6) dan (-2,-2)

KOORDINAT GEOMETRI
JAWAPAN

SABK S2

$$m = \frac{1}{6}$$

$$y - 8 = \frac{1}{6}(x - 3)$$

$$y = \frac{1}{6}x + \frac{15}{2}$$

$$F(x, y) = \left(\frac{3(3) + 1(5)}{1 + 3}, \frac{3(8) + 1(-4)}{1 + 3} \right)$$

$$= \left(\frac{14}{4}, \frac{20}{4} \right)$$

$$= \left(\frac{7}{2}, 5 \right)$$

$$= \frac{1}{2} \begin{vmatrix} 3 & 0 & 7/2 & 3 \\ 8 & 0 & 5 & 8 \end{vmatrix}$$

$$= \frac{1}{2} \left| 3(0) + 0(5) + \frac{7}{2}(8) - 0(0) - 0\left(\frac{7}{2}\right) - 3(5) \right|$$

$$= 6.5 \text{ unit}^2$$

$$6 = \sqrt{(x - 5)^2 + (y - (-4))^2}$$

$$36 = x^2 - 10x + 25 + y^2 + 8y + 16$$

$$x^2 + y^2 - 10x + 8y + 5 = 0$$

SABK S2

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$3(0)^2 - 6(0) - 9 = -9$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$$

$$3(x - 3)(x + 1) = 0$$

$$x = 3 \quad x = -1$$

$$y = (-1)^3 - 3(-1)^2 - 9(-1) + 5 = 10$$

Q(-1, 10)

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$6(-1) - 6 = -12 < 0$$

Maka Q adalah titik maksimum.

$$y = 5^3 - 3(5)^2 - 9(5) + 5 = 10$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

$$= (3(5)^2 - 6(5) - 9) \times \left(\frac{3}{100} \times 5 \right)$$

$$= 5.4$$

$$\frac{\delta y}{y} \times 100 = \frac{5.4}{10} \times 100$$

$$= 54$$

SABK TRIAL

$$\frac{m(7) + n(-3)}{m + n} = 3$$

$$4m = 6n$$

or

$$\frac{m}{n} = \frac{6}{4}$$

$$m : n = 3 : 2$$

$$\frac{3(6) + 2(p)}{5} = 4$$

$$18 + 2p = 20$$

$$p = 1$$

$$\frac{1}{2} | [(0)(1) + (-3)(6) + (7)(9)] - [(-3)(9) + (7)(1) + (0)(6)] |$$

$$\frac{1}{2} | 45 - (-20) |$$

$$\frac{1}{2} | 65 |$$

$$32.5 \parallel 32 \frac{1}{2} \parallel \frac{65}{2} \text{ units}^2$$

MIMS S1

$$m_1 = m_2 = -8$$

Guna (-2, 12)

$$y = -8x - 4$$

M(0, -4)

$$\text{Area} = \frac{1}{2} | [(-2)(-4) + (0)(6) + (-4.5)(12)] - [(0)(12) + (-4)(-4.5) + (6)(-2)] |$$

$$\text{Area} = 26$$

$$2SJ = SM$$

$$2\sqrt{(x+2)^2 + (y-12)^2} = \sqrt{(x-0)^2 + (y+4)^2}$$

$$3x^2 + 16x + 3y^2 - 104y + 576 = 0$$

MIMS S2

$$-\frac{1}{2} \times m_{JM} = -1 \quad m_{JM} = -2 \quad \text{or } J(-4,0)$$

$$y = -2x - 8$$

$$-2x - 8 = \frac{1}{2}x + 6,$$

$$4x + x = -16 - 12$$

$$x = -\frac{28}{5} \quad y = \frac{16}{5}$$

$$M\left(-\frac{28}{5}, \frac{16}{5}\right)$$

Titik tengah LJ(-2,3)

$$\sqrt{(x - (-2))^2 + (y - 3)^2} = \sqrt{(-2)^2 + (3)^2}$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 13$$

$$x^2 + 4x + y^2 - 6y = 0$$

$$\frac{1}{2} \begin{vmatrix} -4 & -\frac{28}{5} & 0 & 4 & -4 \\ 0 & \frac{16}{5} & 6 & 4 & 0 \end{vmatrix}$$

$$\frac{1}{2} \left| \left(-4 \left(\frac{16}{5} \right) + \left(-\frac{28}{5} \right) \times 6 \right) - (6(4) + 4(-4)) \right|$$

$$\frac{1}{2} \left| \left(-\frac{64}{5} - \frac{168}{5} \right) - (24 - 16) \right|$$

$$\frac{136}{5} \text{ unit}^2 // 27.2 \text{ unit}^2 // 27 \frac{1}{5} \text{ unit}^2$$

MIMS S3

$$S(-1,1), T(1,-1)$$

$$WP = 2$$

$$\sqrt{(x - (-1))^2 + (y - (-1))^2} = 2$$

$$x^2 + y^2 + 2x + 2y - 2 = 0$$

$$\text{Titik tengah } AB = \left(\frac{4+n}{2}, \frac{6+t}{2} \right)$$

$$R(1,5)$$

$$m_{RS} = 2 \quad \text{atau} \quad m_{AB} = -\frac{1}{2}$$

$$\frac{\frac{6+t}{2} - 1}{\frac{4+n}{2} + 1} = 2$$

$$\frac{t-6}{n-4} = -\frac{1}{2}$$

$$t = 2n + 8$$

$$t = \frac{16-n}{2}$$

Selesaikan kedua-dua persamaan

$$\frac{16-n}{2} = 2n + 8$$

$$n = 0, t = 8$$

$$B(0,8)$$